Annotation No. 59

EURIBOR AND LIBOR AVERAGES

Euribor, LIBOR, EONIA and Fed Funds. *Interest rate interpolation and extrapolation.*

Brief outline

The Euribor and LIBOR averages are indexes of the interest rates at which international banks are prepared to lend a reasonable sum to each other in Frankfurt and London over a short term of less than one year on an unsecured basis. Euribor and LIBOR averages have a wide practical importance in the financial markets and in the aggregate economic activity: they are used as interest rate references in financial transactions and in various financial instruments. Euribor and LIBOR averages have been improperly set by some banks to portray the financial strength they lacked in the height of the Crisis, and were manipulated as part of a coordinated attempt to earn trading profits in interest rate contracts. The market investigations that have taken place in the U.S., Europe and Japan, have already netted more than 6 billion U.S. dollars in fines and penalties.

EURIBOR, LIBOR, EONIA and *FED FUNDS*

This section is dedicated to a description of these interbank rate measures and how they are calculated. Before we do this, I call your attention to two basic remarks that ought to be present as we analyze the role of these interest rate measures in financial markets.

1. The Euribor and LIBOR averages are not interest rates: they are statistical indexes of interest rate measures. They are (trimmed) averages of the rates related to short-term interbank loans and deposits submitted daily by each panel of international banks in Frankfurt-am-Main and London.
2. The Euribor and LIBOR averages are not interest rate indexes based on real transactions: they are based on indicative interest rate estimates of each bank's borrowing costs at the different funding maturities.

The City of London has been since the late Fifties the preferred international centre by financial institutions for the interbank purchase and sale of short-term cash. In fact, it was in London that the initial excess of liquidity in U.S. dollars arrived in the form of dollar deposits by Russian banks fearing economic retaliation by the U.S. as the Cold War took its first steps. It was also due to the arrival of U.S. dollar balances accumulated by some Indian rich families looking for fiscal protection from the newly independent Indian State. The interbank purchase and sale of dollar balances was also the most elementary form of investing the U.S. dollar liquidity achieved by the European postwar reconstruction. The interbank market for loans and deposits in dollars

outside the U.S. economic territory was then known as the *Eurodollar* market. The prefix "Euro" here means solely "offshore" or "stateless": This was the market for short-term dollars in the "stateless" City of London. The growth of the Eurodollar market accelerated further in the early Seventies as the dollar balances accumulated by the oil exporting states in the Middle East converged to London following the crude price increases that began in 1973. As the Eurodollar market matured, the interbank money-market activities in London were extended to other major currencies such as pounds sterling, Japanese yen, Swiss francs, the dollars from Australia, Canada and New Zealand, the Nordic crowns from Denmark and Sweden, and to the European Common Market composite currency, the *écu*. The City of London was also the "stateless" market for short-term balances denominated in the Continental currencies that joined the Economic and Monetary Union (EMU) up until 1999, like the Portuguese *escudo* and the Spanish *peseta*. After 1999, London has remained an important money-market centre for short-term balances denominated in euros. But Frankfurt, where the headquarters of the European Central Bank (ECB) are located, hass emerged as the reference for the interbank purchase and sale of short-term cash in euros.

The idea of creating a statistical index of the average short-term borrowing costs of the major international banks in London, is generally attributed to Minos ZOMBANAKIS (n. 1929), a Greek banker who headed the London outfit of the Manufacturers Hanover Trust Company.[1] This average rate index was proposed in 1969 as the benchmark for the interest rate in a syndicated loan to the Shah of Persia that was led by Manufacturers Hanover. The growing use of interest rate benchmarks in syndicated loans denominated in Eurocurrencies was part of the launching in 1984 by the British Bankers Association or BBA, of the *LIBOR* averages. LIBOR stands for *London Inter Bank Offered Rate*. The fruitful market experience with the BBA LIBOR averages was the main reason behind the creation of the *Euribor* averages after the introduction of the European single currency under the auspices of the European Banking Federation or EBF. Euribor stands for *EURo Inter Bank Offered Rate*. After its manual-plus-fax beginnings, the daily construction of the BBA LIBOR averages eventually joined the full electronic mode with the help of Thomson Reuters, which has provided the daily administrative data collection from the banks and the calculation of the trimmed averages. After the changes introduced on January 1, 2014, in the wake to the so-called 'scandal' of the BBA LIBOR averages, the London benchmarks began to be administred by a subsidiary of the InterContinental Exchange (ICE) and were officially renamed as ICE LIBOR averages. The calculation of the LIBOR averages is also a regulated financial activity.

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[1] The Manufacturers Hanover Trust Company is an earlier incarnation of what is today the J.P. Morgan Chase & Co., the North-American bank. Cf. David Lascelles. *The Story of Minos Zombanakis: Banking Without Borders* (Athens: 2011) Kerkyra Publications, S.A., pp. 85-86.

Reminder: The Euribor and LIBOR averages.

The following *internet* pages offer information on the construction of the these two interbank money-market rate averages.

- EBF Euribor URL: http://www.euribor.org/.
- BBA LIBOR URL: http://bbalibor.com/.
- ICE LIBOR URL: https://www.theice.com/iba_libor.jhtml/.

My favorite *internet* page with updated information on the Euribor and LIBOR and averages, and with an extensive data archive, is:

- Global Rates URL: http://www.global-rates.com/.

The Global Rates pages are offered in many languages. N.B. The Euribor averages are published with a 24-hour embargo after the daily publication time, since November 2013. ◢

The practical importance of the LIBOR and Euribor averages in financial markets and in the general economic activity is enormous today. They are indexes used as reference rates in financial transactions such mortgage loans, floating rate notes or FRN, and in syndicated loans with a variable rate. They are also the reference rate for derivative contracts such as forward rate agreements on short-term rates or FRA, futures contracts on on short-term rates, and on interest rate swaps. The 3-Month Euribor futures contract traded on the Eurex platform of the Deutsche Börse Group is one of the most heavily traded interest rate futures contract in the World. Some recent estimates on the use of the LIBOR and Euribor averages as interest rate references in financial markets around the World are of about 550 trillion (550×10^{12}) U.S. dollars.[2]

The interest rates quoted in most bank loans today and most mortgage loans in Europe, are described in terms of a *spread* below or above a given LIBOR or Euribor average. For example: The interest rate on a bank loan is often quoted on the basis of an interest rate defined as «*3-Month Euribor minus ¼ percent*». This means that the interest rate must have been set on 25 January 2012 at 0.669 percent minus 0.250 percent. Another example: The interest rate on a mortgage loan could have been defined as «*6-Month Euribor plus 2¼ percent*». This means that the interest rate must have been set on the same 25 January 2012 at 0.669 percent plus 2.250 percent. The magnitude of these spreads depends largely on the perceived default risk of the borrower.

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[2] Cf. David Hou and David Skeie. "LIBOR: Origins, Economics, Crisis, Scandal, and Reform," FRB of New York *Staff Report* No. 667 (March 2014) Federal Reserve Bank of New York.

Example 1

Consider a one-year business loan of 200,000 euros granted on 30 July at a rate defined as 3-Month Euribor plus 2¼ percent a year. Assuming that the 3-Month Euribor average was set at ¾ of one percent, what is the interest amount due at the end of the first interest period (on 30 October)?

Solution. The 3-Month Euribor average is commonly used as a reference in bank loans to small businesses and debt securities issued by multinational corporations in Europe. The actual number of days in the 3/Month period beginning on 30 July is

$$(31 - 30) + 31 + 30 + 30 = 92 \text{ days.}$$

We calculate now the interest amount due based on the periodic rate.

$$200{,}000.00 \times \frac{(2.250 + 0.750)}{100} \times \frac{92}{360} = 1{,}533.3333 \,\text{euros.}$$

The interest amount due on 30 October is 1,533.33 euros. ◢

The methodology used today in the calculation of the LIBOR and Euribor averages is very similar. Each bank participating in each of the LIBOR and Euribor panels of banks is obligated to contribute with an interest rate indication of the level at which it would take an offer for a reasonable nominal amount in the interbank money market for a given currency and a given tenor. The LIBOR averages are constructed on the basis of rate submissions received from banks in response to the following question: "*At what rate could you borrow funds, were you to do so by asking for and then accepting interbank offers in a reasonable size just prior to 11 am London Time?*" After ordering all the rate submissions from the lowest to the highest values, the first and the last quartiles are eliminated, so that the LIBOR reference rate is a truncated average of the middle fifty percent of the rate submissions in each currency denomination and in each tenor prior to 11:00 a.m. in London. (Each Euribor truncated average is calculated on the basis of two-thirds of the rate submissions.) The daily averages are normally set by 11:00 a.m. in each money market center and published in the *internet* pages of the main financial information agencies such as Thomson Reuters or Bloomberg. The LIBOR averages are calculated for seven different tenors in each currency denomination (U.S. dollars, sterling, yen, Swiss francs, and euros) between overnight and one year and are presented as a percentage rate with *five* decimals. The Euribor averages refer to euro denominated interbank loans. They are calculated for seven different tenors and are presented as a percentage rate with *three* decimals. The interbank averages represent in

general a quotation for *spot* value date or two business days (*T* + 2) after the settlement date of the hypothetical interbank loan. The interbank averages make use of the day/year conventions that apply to the currency denomination of the hypothetical interbank loan: The Euribor averages and the U.S. dollar LIBOR averages use the *Actual*/360 method while the yen and sterling LIBOR averages use the *Actual*/365 method. The Swis franc LIBOR averages use the 30/360 method. Table 58.1 shows the Euribor and LIBOR averages as set and published on March 19, 2014.

Please insert here: **Table 58.1** Euribor and LIBOR averages.

The daily LIBOR averages also include EUR LIBOR averages for euro-denominated loans in London, which were justified by the need to ensure the continuity of the rates series associated with the currencies that joined the EMU in 1999. It must however be clear that the EUR LIBOR should not be confused with Deve porém ficar claro que as médias Euro-LIBOR não devem ser confundidas com as Euribor, adoptadas e consideradas como a referência para transacções em euros.

Other money market centers besides London and Frankfurt have also constructed similar interbank rate averages. The TIBOR average which designates the *Tokyo Inter Bank Offered Rate* is the reference for the yen cash markets in Tokyo. The CIBOR average which designates the *Copenhagen Inter Bank Offered Rate* is the reference for the Danish krone in Copenhagen. The SIBOR average which designates the *Stockholm Inter Bank Offered Rate* is the reference for the Swedish krona cash markets in Stockholm. The use of the LIBOR averages and then the Euribor averages has widened well beyond their initial purpose of describing the overall funding cost conditions of international money market banks. LIBOR and Euribor averages are used as interest rate indicators for SME lending, and for residential mortgage pricing, and are the underlying asset of a number of interest rate futures and options contracts worlwide. The Crisis has had a major impact on the calculation procedures and publication of the LIBOR and Euribor averages which in themselves have become regulated financial activities. In particular, the LIBOR averages are no longer arranged and published under the aegis of the BBA which has been replaced by ICE LIBOR Limited.

The EONIA average

Besides the Euribor and USD LIBOR averages, there are two other interbank average rate benchmarks that are most important both economic and financially: the *EONIA* average and the so-called *effective Fed Funds average*. EONIA stands for *Euro Over-Night Index Average*. The EONIA average is a weighted average of the unsecured overnight interbank loans originated by the Euribor panel member banks until 6 p.m. (CET) on a TARGET day. (TARGET stands for.) The *overnight* maturity means the

one-day period from one TARGET day till the next TARGET day. The EONIA average is calculated everyday by the ECB and is ready to be published by 7 p.m. (CET) using the same publishing conventions as the Euribor averages. The EONIA average is normally viewed as the most important money-market benchmark since the value of euro-denominated interest rates for the various maturities depend on it fundamentally.

The efective *Fed Funds* average

The U.S. dollar overnight interbank benchmark that corresponds to the EONIA average is the the so-called *effective Fed Funds average*. *Federal Funds* or *Fed Funds* is the general description of the cash balances held as reserves at the Federal Reserve by each bank participating in the U.S. banking system. The cash takings by a bank in the form of deposits increase the outstanding value of its Federal Funds. Similarly, the cash placings in the form of bank loans reduce the outstanding value of its Federal Funds. The member banks manage their minimum required balances on their reserve accounts by taking and placing funds from and to other banks through transfers over the Federal Reserve Wire system. The taking of Federal Funds in this segment of the U.S. dollar money market is described as a Fed Funds *purchase*; the corresponding placing of Federal Funds is described as a Fed Funds *sale*. Given the high variability of the daily funds requirements by banks, the Fed Funds market is essentially an overnight market for cash. The overnight Fed Funds rate is definitely the most important U.S. dollar money-market benchmark rate.

Other domestic money markets have also their overnight money-market benchmark rates. The overnight rate benchmark for sterling-denominated funds is the *SONIA* average, which is administered by the Bank of England. SONIA stands for *Sterling Over-Night Index Average*.

INTEREST RATE INTERPOLATION AND EXTRAPOLATION

Until very recently, the daily publication of the Euribor and LIBOR averages included the values of all the monthly maturities from one to twelve months. Following the recommendations contained in the Wheatley Report, the Euribor and LIBOR averages are now only calculated for the one-, two-, three-, six-, and twelve-month maturities. Commercial banking often requires the calculation of other standard monthly maturities like the 9-month maturity and of maturities expiring on nonstandard or *broken* dates. [3] The purpose of this section is to offer a simple method to calculate the corresponding

[3] The money markets for the euro and the U.S. dollar also include in their standard maturities those ending on the so-called *IMM dates*. The IMM dates are the expiry dates of the interest rate futures contracts traded on the International Money Market, in Chicago.

annual interest rates for nonstandard maturities ending within the money-market horizon and even slightly outside of it like the maturity of one-year from today plus a few days.

Suppose two standard maturities ending on dates t_1 and t_2 such that $t_1 \leq t_2$. The number of actual days between these two dates on the basis of either the *ACT*/360 convention or the *ACT*/365 convention equals the difference $t_2 - t_1$ measured in days. Consider now an intermediate date t described as a *broken* date, which is relevant to some banking transaction. We have that: $t_1 \leq t \leq t_2$. Suppose that the values of the annual interest rates $r(0, t_1)$ and $r(0, t_2)$ are known. Our goal here is to offer a simple method to calculate the value of the annual rate $r(0, t)$ based on the known values of those two annual rates.

The central idea of a *linear interpolation* is to take account of the daily change in the annual rate between two values of the annual rate within the time period defined by the two known maturity dates. Based on the daily change in the annual rate between the two known maturity dates t_1 and t_2, it is only required to add the implied total change in the annual rate between the dates t_1 and t to the known value of $r(0, t_1)$.

$$r(0, t) = r(0, t_1) + \frac{r(0, t_2) - r(0, t_1)}{t_2 - t_1} \cdot (t - t_1) .$$

Please note that the difference $(t_2 - t_1)$ is equal to: $(t_2 - t) + (t - t_1)$. Following some simple transformations, we can obtain a general expression yielding the linearly interpolated value for the intermediate annual rate $r(0, t)$.

$$r(0, t) = \frac{r(0, t_2) \cdot (t - t_1) + r(0, t_1) \cdot (t_2 - t)}{t_2 - t_1} .$$

This general expression may also be re-written as follows:

$$r(0, t) = r(0, t_2) \cdot \frac{t - t_1}{t_2 - t_1} + r(0, t_1) \cdot \frac{t_2 - t}{t_2 - t_1} .$$

The linearly interpolated value of the annual rate is a time-weighted average of the known annual interest rate values.

Most computing devices can also be applied to non-linear interpolation techniques. These techniques make use of smooth interpolation curves instead of straight lines, which allow for the calculation of differential measures besides the gradient. Among these non-linear techniques, the polynomial interpolation seemed to be a promising technique in spite of the more intensive computing requirements associated with higher order polynomials. The so-called *spline* techniques consisting of segmented low-order (cubic) polynomials are popular today along with the recent *wavelet* techniques.

Example 2

Consider the following USD LIBOR averages that were set on April 15 (2013), in percentage terms:

1 M	3 M	6 M	9 M	12 M
0.19970	0.27760	0.43790	0.57470	0.71900

Please note that the daily publication of the USD LIBOR 9M average was discontinued on November 1 (2013). Suppose that the 9-month (or 275 days) average is not known and that you are asked to show the corresponding value by linear interpolation.

Solution. We use the general expression above to calculate the requested value and plug in it the known values for the 6-month (183 days) and 12-month (365 days) maturities.

$$r(0, 275) = r(0, 365) \cdot \frac{275 - 183}{365 - 183} + r(0, 183) \cdot \frac{365 - 274}{365 - 183}.$$

The 9-month annual rate obtained by linear interpolation from the known values for the 6-month and 12-month maturities is: $r(0, 275) = 0.579945$ percent. The value obtained by linear interpolation is higher than the set value of 0.57470 percent. ◢

The other technical and practical necessity that we are often confronted with is that of a *linear extrapolation*. In this case, we look for a corresponding value associated with a date that is beyond the time horizon of our knowledge. In some cases, this may call for a money-market maturity date of a few days beyond the one-year maturity.

Suppose that we know the value of the annual rate $r(0, t_1)$ and are requested to estimate the value of the annual rate $r(0, t)$ whose maturity period ends on date t such that $t_1 \leq t$. The central idea of linear extrapolation is to use the daily interest accrual rate until the date t_1 in order to extend it over the time interval between the dates t_1 e t.

$$r(0, t) = r(0, t_1) + \frac{r(0, t_1)}{t_1} \cdot (t - t_1).$$

A simple expression describing the relevant linear extrapolation can be obtained through a similar algebraic transformation.

$$r(0, t) = r(0, t_1) \cdot \frac{t}{t_1}.$$

The annual rate corresponding to the nonstandard extended maturity is an extended annual rate of the longest known statdard maturity.

Example 3

Consider the same USD LIBOR averages set on April 15 (2013), in percentage terms, that were indicated in the previous example. Suppose now that you are asked to show the corresponding value of the annual rate for the 13-month (or 395 days) maturity by linear extrapolation.

Solution. Likewise, we use the general expression and plug in it the known value of the USD LIBOR 12M average. This 12-month maturity has 365 days.

$$r(0, 395) = r(0, 365) \cdot \frac{395}{365}.$$

You may be able to confirm that the linearly extrapolated value of the 13-month annual rate is: $r(0, 395) = 0.77809589$ percent. ◢

Key concepts

interpolation, interpolación, interpolação, interpolation, interpolation, interpolazione

extrapolation, extrapolación, extrapolação, extrapolation, extrapolation, estrapolazione

Questions

1. Explain what a linear interpolation is.
2. Explain what a linear extrapolation is.
3. Explain what you think a non-linear interpolation is.

Exercises

1. Based on the data from Table 58.1, suppose that you are requested to show the corresponding value for the Euribor 9M average.
2. Based on the data from Table 58.1, suppose that you are requested to show the corresponding value for the JPY LIBOR 9M average.
3. Based on the data from Table 58.1, suppose that you are requested to show the corresponding value for the Euribor average whose maturity period ends on August 19 (2014).

4. Based on the data from Table 58.1, suppose that you are requested to show the corresponding value for the USD LIBOR average whose maturity period ends on August 19 (2014).
5. Based on the data from Table 58.1, assume that the Euribor 6M average is not known. Calculate the corresponding value for the Euribor 6M average and compare it with the actual value.
6. Based on the data from Table 58.1, assume that the USD LIBOR 12M average is not known. Calculate the corresponding value for the USD LIBOR 12M average and compare it with the actual value.

N.B. All corrections and suggestions or comments to this version of the text are gratefully acknowledged in advance. Thank you.

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Table 58.1 Euribor and LIBOR averages (March 19, 2014).

	Euribor	**LIBOR**				
		CHF	**EUR**	**GBP**	**JPY**	**USD**
O/N	–	–	0,12143	0,47000	–	0,08900
S/N	–	- 0,00200	–	–	0,06214	–
1W	0,195	- 0,01000	0,16357	0,46750	0,07857	0,12300
2W	0,210	–	–	–	–	–
1M	0,237	- 0,00700	0,21286	0,48438	0,10214	0,15755
2M	0,274	0,00800	0,22500	0,17500	0,12357	0,20000
3M	0,312	0,02499	0,28000	0,25000	0,14000	0,23385
6M	0,414	0,07840	0,37271	0,33500	0,19143	0,32970
12M	0,585	0,19940	0,54886	0,63167	0,34857	0,55550

Note: O/N: overnight; S/N: spot/next; 1W: one week; 1M: one month.

Source: Global-Rates.com; and, FRB of St. Louis, FRED Database.

Table 58.2 The IBOR 'scandal' timeline.

2007 **August 9:** BNP Paribas stops publication of unit prices of two mortgage-backed securities funds because of unreliable calculations.

August 10: U.S. Federal Reserve, ECB and Bank of Japan, coordinate massive $266 billion injection of funds to manage liquidity in interbank markets.

September 3: "Barclays Takes a Money-Market Beating," (Mark Gilbert, Bloomberg).

September 13: Bank of England has secretly given liquidity support to Northern Rock (BBC News). Bank run on the next day follows.

September 18: Federal Reserve lowers Fed Funds target to 1 per cent from 5 per cent, one year earlier.

September 25: (Gillian Tett, FT)

November 15: Barclays makes a credit write-down of $2.6 billion.

2008 **April 14:**

April 16: (Carrick Mollenkamp, WSJ)

May 29: "Study Casts Doubt on Key Lending Rate: Banks May Have Filed Flawed Interest Rate Data for Libor Benchmark," (Carrick Mollenkamp and Mark Whitehouse, WSJ, A1)

September 15: Lehman Brothers seeks protection under Chapter 11.

September 29: The U.K. nationalizes Bradford & Bingley, a mortgage bank, and Bank of England extends $32.5 billion credit line to the bank.

2011 **December 9:** Japanese SESC fines Citi for manipulating Euroyen TIBOR.

2012 **June 27:** "Barclays fined a record £290m," (Brooke Masters, Caroline Binham and Kara Scannell, FT).

July 18: "Rate probe turns to four major banks," (FT Online)

July 19: "Barclays the biggest Libor liar? No, that may have been Citi," (Stephen Gandel, Fortune/CNN Money).